Contact Stability for Two-Fingered Grasps

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Abstract

We distinguish between two types of grasp stability, which we call spatial grasp stability and contact grasp stability. We show via examples that spatial stability cannot capture certain intuitive concepts of grasp stability and hence that any full understanding of grasp stability must include contact stability. We derive a model of how the positions of the points of contact evolve in time on the surface of a grasped object in the absence of any external force or active feedback. From this model, we obtain a general measure of the contact stability of any two-fingered grasp. Finally, we discuss the consequences of this stability measure and a related measure of contact manipulability on strategies for grasp selection.

1 Introduction

Stability is the tendency of a system to return to an equilibrium state when displaced from this state. Currently, there are two different views of what constitutes the state of a grasp, leading to two different concepts of grasp stability. The first views the state as the position (and velocity) of the grasped object relative to the palm of the hand. Hence, grasp stability refers to the tendency of the object to return to its original position when displaced by an outside force. We call this *spatial grasp stability.* The second views the state as the position (and velocity) of the points of contact on the surfaces of the object and the fingers. Hence, grasp stability is the tendency of the points of contact to return to their original locations in response to a disturbance. We call this *contact grasp stability.*

1.1 Past research on grasp stability

Both of these types of grasp stability have been investigated in the past. For spatial grasp stability, a key concept is that of force closure, which is the ability of the fingers to apply arbitrary force and torque to the object [20, 7]. (Note that Salisbury [20] originally called this "form closure".) For force closure grasps, the system (whose state is the position and velocity of the object) is locally controllable and can be stabilized by an appropriate control law. Nguyen [18] discusses a class of control laws which stabilize a grasp in this sense.

The problem with the force-closure criterion from the viewpoint of grasp selection is that it only categorizes a grasp as spatially stable or unstable and does not distinguish among the generally infinite number of stable grasps. To make this distinction, there have been different criteria proposed, which fall into two categories. Members of the first category measure how easily (i.e., with how small a torque at each of the joints) the fingers can oppose arbitrary forces and torques and thus stabilize the system [7, 9, 19]. While the definition of a single number summarizing many degrees of freedom can vary, all such measures use as their basic information the configuration of the fingers and the locations of the points of contact.

Members of the second category measure the size of a buffer between the current grasp and the nearest spatially unstable grasp. Kerr and Roth [8] consider the buffer zone to be the friction cone at each point of contact and the stability measure as the smallest angle between the line connecting the two points of contact and the edge of the friction cone. Nguyen [17] considers the buffer zone to be the area around each point of contact through which that point of contact can move and still form a force closure grasp with the other point of contact. The grasp stability is the minimum distance from any point of contact to the edge of its buffer.

Nguyen's measure captures an important aspect of grasp stability, but it is missing a crucial piece, which is how easily the points of contact move. If the buffer is small but the points of contact are virtually glued to their current positions, then there is no danger of losing force closure via motion of the points of contact. Conversely, if the buffer is large but the points of contact move around easily, then the grasp is unstable in this sense. In Section 6, we introduce the concept of contact manipulability, which along with contact stability allows us to formalize the notion of how easily the points of contact move.

There are also two categories of work on contact stability, those investigating sliding contact and those examining rolling contact. For sliding contact, Hanafusa and Asada [6] built a hand whose fingertips slid along the surface of a grasped object in such a way as to minimize a mechanical potential energy. Hence, in response to any small displacement of the contact points, the system would return to its equilibrium position, and the grasp would therefore be (contact) stable. Nguyen [18] discusses a similar approach to stability with sliding contact. The problem with sliding contact is that without static friction spatial stability generally requires at least seven points of contact as compared to two points of contact for soft fingertips with friction [17].

The alternative to sliding contact is rolling contact. The primary past work on contact stability with rolling contact is by Cutkosky and Wright [4]. They analyze different two-fingered grasp scenarios to determine the contact stability of each. The factors which they found to contribute to grasp stability are exactly those which we illustrate through the examples of Section 2: (1) local geometry of the object at the points of contact, (2) local geometry of the fingertips, (3) distance between the points of contact, (4) viscoelasticity of the fingers and object, (5) applied normal force, and (6) moment of inertia of the grasped object. What Cutkosky and Wright did not do is derive a general measure of contact stability for an arbitray two-fingered grasp; a derivation of such a measure is the primary result of this paper.

1.2 Summary of our work

This paper is an extended and improved version of [15]. The remainder of this paper is organized as follows. Section 2 illustrates using examples the importance of the six factors for contact stability mentioned above. These examples also show that there are some obvious differences in stability which cannot be captured via measures of spatial stability, thus highlighting the importance of contact stability.

Section 3 discusses contact kinematics, which is the relation between the relative motion of two contacting objects and the motion of their point of contact across their surfaces [13, 2]. The local geometry of the objects (in particular their curvatures), play a key role in contact kinematics, and this is how the local geometries of the fingers and object enter into our analysis of grasp stability.

In Section 4, we derive the central results of the paper: a model of the natural dynamics of the points of contact of a two-fingered grasp and a measure of contact stability based on this model. To do this, we first calculate the torque exerted on the object at a point of contact due to the motion of this point. This torque is the sum of three components, which we call the attractive torque, the repulsive torque, and the dissipative torque. Torque exerted at **one** point of contact causes angular acceleration of the object and thus causes acceleration of**both** points of contact relative to the object. This coupling of the motions of the points of contact means that we cannot consider each point of contact separately but instead must treat the grasp as a complete system. (We call the component of acceleration of a point of contact caused by a torque at that point of contact the "local acceleration" and the component of acceleration caused by a torque at the other point of contact "remote acceleration".) For a two-fingered grasp, this system

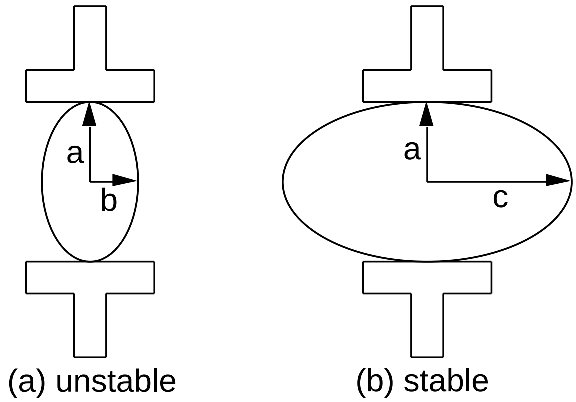

Figure 1: The local geometry of the object affects stability.

is eight-dimensional (two degrees of freedom for the position of each point of contact and two degrees of freedom for the velocity of each point of contact), and the state can therefore be represented as an 8-vector $\vec{g}$. Substituting the expressions for the torques at each point of contact into Newton's Law for angular acceleration gives an equations of the form $\dot{\vec{g}} = A\vec{g}$, where A is an 8x8 matrix which is a function of the six factors mentioned above which contribute to grasp stability. This system is stable if and only if the eigenvalues of A lie in the left half-plane. A measure of grasp stability is the maximum of the real parts of these eigenvalues.

In Section 5, we examine an important class of grasps for which the eigenvalues of A have a relatively simple closed-form representation. For this class of grasps, we show how our measure of contact stability is consistent with all the intuitions about contact stability set forth in the examples of Section 2.

Finally, Section 6 discusses the consequences of our measure of contact stability for grasp selection. We introduce the concept of contact manipulability, defined loosely as the gain of the map from the rolling velocity at a point of contact to the velocity of that point. We show how considerations of contact stability and manipulability affect our selection not only of grasp points on the object but also of the location of contact points on the fingers, the matching of fingers to grasp points, and the orientation of the object relative to the fingers.

2 Aspects of grasp stability

In this section, we examine examples which illustrate certain important aspects of grasp stability and why measures of spatial stability cannot capture them.

Example 1 Consider two ellipsoids, one given by the equation $x^2/a^2 + y^2/b^2 + z^2/b^2 = 1$ and the other by the equation $x^2/a^2 + y^2/c^2 + z^2/c^2 = 1$, where $b < a < c$. Consider grasping these ellipsoids with flat fingertips at the points $(a, 0, 0)$ and $(-a, 0, 0)$. (Figure 1 shows a cross-section of these grasps.) The second grasp is more stable than the first. This example illustrates the importance of the local geometry of the grasped object to grasp stability.

Note that the class of spatial stability measures which consider the ease of force application would consider these two grasps equivalent because the finger configurations and the grasp points are the same in both grasps. Hence, this class of stability measures is incomplete. The other class of spatial stability measures discussed in Section 1.1, those which consider buffer zones, would correctly identify the second grasp as more stable (but will not correctly identify the more stable grasps in Examples 2 and 5).

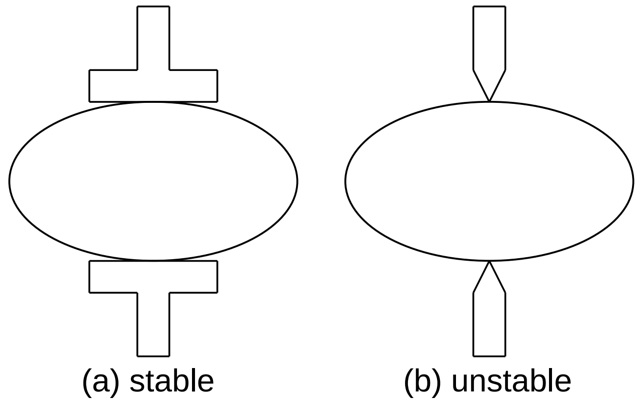

Figure 2: The local geometry of the fingers affects stability.

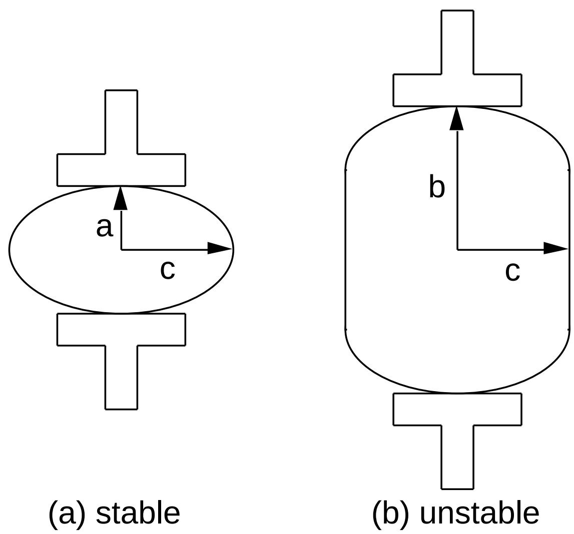

Figure 3: The distance between the points of contact affects stability.

Example 2 Consider two different grasps identical in all respects except the shape of the finger surfaces. In one case the fingers are flat, while in the other case the fingers are pointed. (Figure 2 illustrates these two scenarios.) The first grasp is more stable than the second. Hence, the local geometry of the fingers affects grasp stability.

Note that spatial stability measures of both the "ease-of-force-application" and "buffer-zone" classes cannot distinguish between these two grasps because they both ignore the one distinguishing factor, the local geometry of the fingers.

Example 3 Consider two objects, one an ellipsoid given by the equation $x^2/a^2 + y^2/c^2 + z^2/c^2 = 1$ and the second obtained by cutting this ellipsoid in half and inserting between the two halves an ellipsoidal cylinder of height $2b - 2a$, where $a < c < b$ (see Figure 3). Consider grasping these objects with flat fingertips, and let the grasp points be $(a, 0, 0)$ and $(-a, 0, 0)$ in the first case and $(b, 0, 0)$ and $(-b, 0, 0)$ in the second case. The first grasp is more stable than the second despite the objects and the fingers having the same local geometry because the points of contact are closer to each other in the first case than in the second case.

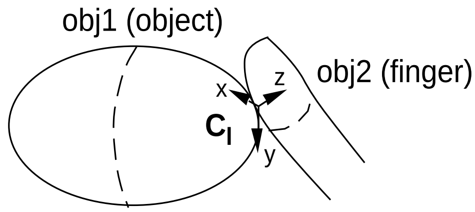

Figure 4: A local reference frame for a point of contact.

Example 4 Consider two unstable grasps (such as the one pictured in Figure 1a) which are identical in all ways except the material properties of the fingers. In the first case, the finger surface is either rigid or purely elastic, while in the second case the finger surface is viscoelastic, or in more common language "soft" or "squishy". In the second case the grasp is less unstable [1, 4].

Example 5 Consider the unstable grasp of Figure 1a, and consider two different situations. In the first, the fingers exert relatively small normal forces, while in the second, the fingers exert large normal forces. All else being equal, the latter is more unstable, a phenomenon known as the "coin-snap problem" [4].

Note that spatial stability measures of both the "ease-of-force-application" and "buffer-zone" classes again cannot distinguish between these two grasps.

Example 6 Consider the unstable grasp of Figure 1a, and consider two different situations in which the grasped object has two different masses. All else being equal, the grasp with the lighter object is more unstable. We will see that the mass of the grasped object affects grasp stability via its effect on the object's moment of inertia.

We have thus identified six factors which seem intuitively to be important in grasp stability: (1) local geometry of the object, (2) local geometry of the finger, (3) distance between the points of contact, (4) finger and object viscoelasticity, (5) applied normal force, and (6) object mass. We will below derive a measure of grasp stability which incorporates all of these factors. However, we first review contact kinematics.

3 Contact kinematics

In the nomenclature of [13], *contact kinematics* refers to the evolution of a point of contact on the surfaces of two objects in response to a relative motion of these objects. We now present in a simplified form some concepts and equations from the study of contact kinematics relevant to our analysis of grasp stability.

Consider two rigid objects, obj_1 and obj_2, which have a single point of contact (see Figure 4).

Definition 1 A *local reference frame* for this point of contact is a right-handed, orthonormal coordinate frame whose origin is at the point of contact and whose z axis is the outward normal to obj_1 at this point. (The x and y axes can be any unit vectors satisfying the constraints of orthogonality and right-handedness.)

Definition 2 The *curvature* of a surface S at a point $\vec{s} \in S$ relative to two unit vectors $\vec{x}$ and $\vec{y}$, which are orthogonal to each other and tangent to S at $\vec{s}$, is the 2x2 matrix K such that any infinitesimally small displacement $\vec{\Delta s} = [\Delta s_x, \Delta s_y, 0]^T$ of $\vec{s}$ along S results in a change of $\vec{n}$, the outward normal to S, of

$$\vec{\Delta n} = \vec{n}(\vec{s} + \vec{\Delta s}) - \vec{n}(\vec{s}) = \begin{bmatrix} K \begin{bmatrix} \Delta s_x \\ \Delta s_y \end{bmatrix} \\ 0 \end{bmatrix} \tag{1}$$

The *principle curvatures* are the eigenvalues of the curvature matrix. (Note that the principle curvatures are always real because the curvature matrix is symmetric.) The *principle axes of curvature* are the directions on the surface corresponding to the eigenvectors of the curvature matrix. (Note that the principle axes are always orthogonal because the curvature matrix is symmetric.)

Let C_l be the chosen local reference frame for $obj1$ and $obj2$, and let the x and y directions be defined by the coordinate axes of C_l. Let $\dot{\vec{s}}_1 = [\dot{s}_{1x}, \dot{s}_{1y}, 0]^T$ and $\dot{\vec{s}}_2 = [\dot{s}_{2x}, \dot{s}_{2y}, 0]^T$ be the rate at which the point of contact moves across the surfaces of obj_1 and obj_2 respectively. Let $\vec{v} = [v_x, v_y, v_z]^T$ and $\vec{\omega} = [\omega_x, \omega_y, \omega_z]^T$ be the translational and rotational velocities of $obj1$ relative to $obj2$, where the reference frame for $obj1$ is the local reference frame. Let K_1 and K_2 be the curvatures of $obj1$ and $obj2$ at the point of contact relative to the x and y axes of the local reference frame. Call the quantity $K_r = K_1 + K_2$ the *relative curvature* at the point of contact. The *contact constraint* $v_z = 0$ is a necessary and sufficient condition that the objects remain in contact. We can then derive the *contact equations*

$$\dot{\vec{s}}_1 = K_r^{-1}(\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} - K_2 \begin{bmatrix} v_x \\ v_y \end{bmatrix}), \quad (2)$$

$$\dot{\vec{s}}_2 = K_r^{-1}(\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} + K_1 \begin{bmatrix} v_x \\ v_y \end{bmatrix}) \quad (3)$$

For two different derivations of these equations, see [13] and [2].

We note the following about the contact equations:

1. When we allow the point of contact to move more than an infinitessimally small amount across the object surfaces, the contact equations in the form given above are no longer valid. For a globally valid formulation, see [13]. For the purposes of this paper, the "local" formulation is sufficient.

2. The derivations of the contact equations is based on the assumption that the objects are rigid. However, below we will allow the objects to be viscoelastic. Experimental results and theoretical analysis [12, 13] indicate that the contact equations remain valid for such objects as long as the motion is pure rolling, i.e. $v_x = v_y = \omega_z = 0$. In the analysis below, we indeed restrict the motion to be pure rolling.

4 A model of grasp dynamics

We now build a model of how the system (defined as the positions and velocities of the points of contact on the object surface) evolves in response to small disturbances from equilibrium.

4.1 Assumptions

The model utilizes the following underlying assumptions:

(1) There are exactly two points of contact (actually areas of contact due to the viscoelasticity of the fingertip), one for each of two fingers grasping the object.

(2) The fingertips maintain their grasp of the object, i.e. maintain static friction at all point of contact. Because static friction prevents slippage, the object can roll but not slip at the points of contact. (In the notation of Section 3, $v_x = v_y = \omega_z = 0$.) Any force or torque on the object which would cause slippage at a point of contact is compensated for either by an opposing frictional force and/or by a motion of the fingertip to keep it alligned with the object.

(3) The evolution of the points of contact follow Equations 2 and 3. (The validity of this assumption is discussed in Section 3.)

(4) Disturbances from equilibrium are small enough that all first-order approximations are valid, e.g. that the curvatures of the object and fingertip at a displaced point of contact are the same as at the equilibrium point. We also ignore torques which are higher than first order functions of the motion (e.g., Coriolis effects).

(5) The force and magnitude exerted by a fingertip is of constant magnitude and direction relative to its local reference frame. Therefore, as the fingertip rotates relative to the object, the direction of the applied force rotates likewise. Note that tactile sensing can tell us the position of the point of contact; hence, we could make the applied force a function of this tactile feedback in order to stabilize the system. However, for the purposes of this paper, we are interested in the natural stability of the system, i.e. the stability in the absence of any feedback response.

(6) The relative motion of the fingertips has a full six degrees of freedom, which means that the fingers have a combined total of at least six degrees of freedom. (Human fingers each have four degrees of freedom giving a total of eight degrees of freedom for any pair.) [13] shows that this permits realizations of any pair of rolling velocities at the points of contacts. Note that this assumption means that our analysis does not apply to non-dextrous robotic grippers.

4.2 The Generated Torques

The torque generated by the motion of a point of contact across the object surface is the sum of three components, which we call (1) the attractive torque, (2) the repulsive torque, and (3) the dissipative torque. (We describe the origin of each of these and derive its value. Note that there are other torques at a point of contact due to factors such as static frictional forces and compliance. However, because these torques are not functions of the position and/or velocity of any point of contact, they have no relevance to the stability of these points. They instead are just a source of disturbance on the system.)

Attractive torque: An attractive torque arises from the change in position at which a finger applies force. If the point of contact moves a small amount $\vec{\Delta s} = [\Delta s_x, \Delta s_y]^T$ along the object's surface, then the vector from the object's center of mass to the point of contact changes by $\vec{\Delta r} = [\Delta s_x, \Delta s_y, 0]^T$ (measured relative to the object's contact frame). This produces an additional torque around the center of mass of

$$\tau_a = \vec{\Delta r} \times \vec{F} = \begin{bmatrix} \Delta s_x \\ \Delta s_y \\ 0 \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \tag{4}$$

where $\vec{F}$ is the force exerted by the finger on the object. Using Equation 2, we can rewrite this as

$$\begin{aligned} \tau_a &= \begin{bmatrix} K_r^{-1} \begin{bmatrix} -\Delta\theta_y \\ \Delta\theta_x \end{bmatrix} \\ 0 \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \\ &= \begin{bmatrix} F_z (J^T K_r^{-1} J) \begin{bmatrix} \Delta\theta_x \\ \Delta\theta_y \end{bmatrix} \\ \tau_{az} \end{bmatrix} \end{aligned} \tag{5}$$

where K_r is the relative curvature at the point of contact, $\vec{\Delta\theta} = [\Delta\theta_x, \Delta\theta_y, 0]^T$ is the angular displacement (of the object relative to the finger) from equilibrium, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, and τ_{az} is a quantity about whose value we do not care because we have assumed that friction will compensate for this torque.

Note that (1) K_r always has positive, real eigenvalues due to physical constraints and hence so does K_r^{-1}, (2) $J^T K_r^{-1} J$ has the same eigenvalues as K_r^{-1}, and (3) F_z is always negative due to physical constraints.

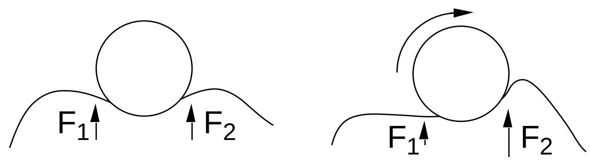

Figure 5: The origin of the dissipative torque.

So, $F_z(J^T K_r^{-1} J)$ has negative, real eigenvalues, and it therefore always acts to oppose displacement from equilibrium in each of its principle directions. Hence, the torque is attractive.

Repulsive torque: The repulsive torque arises from a change in the direction of the applied force. With the assumption that the force is always applied in the same direction relative to the fingertip, the change in the direction of the applied force is $-\vec{\Delta\theta}$; therefore, the change in the force is $\Delta\vec{F} = -\vec{\Delta\theta} \times \vec{F}$. This produces an additional torque of

$$\begin{aligned}
\tau_r &= \vec{r} \times (-\vec{\Delta\theta} \times \vec{F}) = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \times (-\begin{bmatrix} \Delta\theta_x \\ \Delta\theta_y \\ 0 \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}) \\
&= \begin{bmatrix} \begin{bmatrix} -r_zF_z - r_yF_y & r_yF_x \\ r_xF_y & -r_zF_z - r_xF_x \end{bmatrix} \begin{bmatrix} \Delta\theta_x \\ \Delta\theta_y \end{bmatrix} \\ \tau_{rz} \end{bmatrix} \qquad (6)
\end{aligned}$$

where τ_{rz} is a quantity about which we do not care.

Note that the eigenvalues of the matrix $\begin{bmatrix} -r_zF_z - r_yF_y & r_yF_x \\ r_xF_y & -r_zF_z - r_xF_x \end{bmatrix}$ are $-r_zF_z$ and $-(r_xF_x + r_yF_y + r_zF_z) = -\vec{r} \cdot \vec{F}$. For most grasps, these eigenvalues are positive, and hence the torque is repulsive. However, there do exist situations for which one or both of these eigenvalues are negative, and hence the torque is partially or fully attractive, thus belying its name.

Dissipative torque: The dissipative torque arises from the viscoelasticity of the fingers. This viscoelasticity produces a resistance to rolling, i.e. a torque around the point of contact which opposes the relative rotation of the two objects. Cutkosky and Wright [4] introduced rolling resistance into the robotics literature, acknowledging that it had previously been discussed at length in the literature on wheels and tires (see, e.g. [5]). We model the resulting torque on the object as

$$\tau_d = -\kappa_f F_n[\omega_x, \omega_y, 0]^T = \kappa_f F_z[\dot{\Delta\theta}_x, \dot{\Delta\theta}_y, 0]^T \qquad (7)$$

where $F_n = -F_z$ is the normal force and κ_f is a constant called the *coefficient of rolling resistance* [5].

Figure 5 shows how this torque arises. In Figure 5, the top object is rigid, and the bottom object consists of an elastic membrane containing a viscous fluid (like the fingertip described by Brockett [1]). When the objects remain stationary, as picture in Figure 5a, the pressure from the fluid acts evenly over the area of contact, thus producing no net torque. However, when the top object rolls across the bottom object, pressure builds on the side to which the top object is rolling. This imbalance of pressure creates a net torque, which opposes the rolling motion and which is an increasing function of the rolling velocity. Equation 7 provides a first order approximation to the value of the induced torque.

4.3 The Equations of Motion

Let C_1 be a local reference frame for the grasped for the grasped object at one (equilibrium) point of contact and C_2 be a local reference frame at the other (equilibrium) point of contact (see Figure 6). Let $(\vec{p}_0, R_0)$

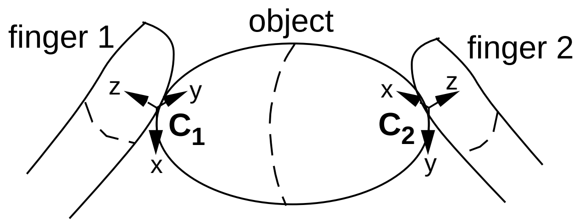

Figure 6: An object grasped by two fingers.

be the rigid-body transformation which takes C_1 to C_2. Let $\tau_1 = \tau_{a1} + \tau_{r1} + \tau_{d1}$ and $\tau_2 = \tau_{a2} + \tau_{r2} + \tau_{d2}$ be the generated torques at the two points of contact. The local effect of these torques (i.e. the effect at the point of contact at which they are applied) is to generate angular accelerations $\ddot{\vec{\Delta}}\theta_{1L} = [\Delta\ddot{\theta}_{x1L}, \Delta\ddot{\theta}_{y1L}]^T$ and $\ddot{\vec{\Delta}}\theta_{2L} = [\Delta\ddot{\theta}_{x2L}, \Delta\ddot{\theta}_{y2L}]^T$

$$\ddot{\vec{\Delta}}\theta_{1L} = I_{23} M^{-1} \mathbf{diag}(1,1,0)\tau_1 = M_{22} I_{23} \tau_1 \tag{8}$$

$$\ddot{\vec{\Delta}}\theta_{2L} = I_{23}(R_0^T M^{-1} R_0)\mathbf{diag}(1,1,0)\tau_2 = \tilde{M}_{22} I_{23} \tau_2 \tag{9}$$

where $I_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $M_{22} = I_{23} M^{-1} I_{23}^T$, $\tilde{M}_{22} = I_{23} R_0^T M^{-1} R_0 I_{23}^T$, and M is the inertia matrix of the object relative to a coordinate frame that is oriented the same as C_1 and whose origin is the center of mass of the object. (Recall from above that the z components of the torques are compensated for by friction.) The torques also cause angular acceleration at the other points of contact. The remote angular accelerations are

$$\ddot{\vec{\Delta}}\theta_{1R} = R_{22}\ddot{\vec{\Delta}}\theta_{2L}, \quad \ddot{\vec{\Delta}}\theta_{2R} = R_{22}^T \ddot{\vec{\Delta}}\theta_{1L} \tag{10}$$

where $R_{22} = I_{23} R_0 I_{23}^T$. Then, the total angular accelerations are

$$\ddot{\vec{\Delta}}\theta_1 = \ddot{\vec{\Delta}}\theta_{1L} + \ddot{\vec{\Delta}}\theta_{1R}, \quad \ddot{\vec{\Delta}}\theta_2 = \ddot{\vec{\Delta}}\theta_{2L} + \ddot{\vec{\Delta}}\theta_{2R}, \tag{11}$$

Substituting the values of the torques derived in Section 4.2 and writing the equations in matrix form, we get

$$\frac{d}{dt}\vec{g} = A\vec{g} \tag{12}$$

where $\vec{g} = [\Delta\theta_{x1}, \Delta\theta_{y1}, \Delta\theta_{x2}, \Delta\theta_{y2}, \Delta\dot{\theta}_{x1}, \Delta\dot{\theta}_{y1}, \Delta\dot{\theta}_{x2}, \Delta\dot{\theta}_{y2}]^T$ and where A is the 8x8 matrix

$$A = \begin{bmatrix} 0 & 0 & I_{22} & 0 \\ 0 & 0 & 0 & I_{22} \\ A_1 & R_{22}A_2 & A_3 & R_{22}A_4 \\ R_{22}^T A_1 & A_2 & R_{22}^T A_3 & A_4 \end{bmatrix} \tag{13}$$

where I_{22} is the 2x2 identity matrix and

$$\begin{aligned} A_1 &= M_{22}(F_{z1} J^T K_{r1}^{-1} J - r_{z1} F_{z1} I_{22} + [r_{y1}, -r_{x1}]^T [-F_{y1}, F_{x1}]) \\ A_2 &= \tilde{M}_{22}(F_{z2} J^T K_{r2}^{-1} J - r_{z2} F_{z2} I_{22} + [r_{y2}, -r_{x2}]^T [-F_{y2}, F_{x2}]) \\ A_3 &= \kappa_f F_{z1} M_{22} \\ A_4 &= \kappa_f F_{z2} \tilde{M}_{22} \end{aligned} \tag{14}$$

4.4 The stability measure

The system described by Equation 12 is stable if and only if the eigenvalues of the matrix A all lie in the left half-plane [11]. Hence, we have derived the condition for contact grasp stability; a grasp is contact stable if and only if the eigenvalues of A lie in the left half-plane.

However, as discussed in Section 1.1 for spatial stability, just classifying a grasp as either stable or unstable is insufficient for comparing two grasps, which might both be stable or both be unstable. So, we define a measure of grasp stability as

$$\max\{Re(\lambda_1), ..., Re(\lambda_8)\} \tag{15}$$

For stable grasps, this measures how fast disturbances are naturally damped out. For unstable grasps, this measures the difficulty of actively stabilizing the grasp. To stabilize a system of greater natural instability requires higher sample rates and/or more precise measurement and/or more precise actuation. Additionally, for fixed sampling rates and measurement and actuation capabilities, a grasp of lesser natural instability will result in faster damping of disturbances than a grasp of greater natural instability after they have both been stabilized.

5 Stability for a class of grasps

In this section, we start by defining an important class of grasps. We derive a closed-form expression for the grasp stability of members of this class. We then show how this stability measures is consistent with the intuitions of Section 2.

5.1 Definition of the class

The grasps we will consider are those satisfying the following conditions:

- $\vec{F}_1 = \vec{F}_2 = [0, 0, -F_n]^T$ (i.e., the forces at the points of contact are of equal magnitude and have no tangential components)
- $\vec{p}_0 = [0, 0, d]^T$ and $R_0 = \begin{bmatrix} R_{22} & 0 \\ 0 & -1 \end{bmatrix}$ where $R_{22} = \begin{bmatrix} \cos\psi & \sin\psi \\ \sin\psi & -\cos\psi \end{bmatrix}$ for some ψ (i.e., the points of contact are diametrically opposed)
- $\vec{r}_1 = [0, 0, r_1]^T$ and $\vec{r}_2 = [0, 0, r_2]^T$ (i.e., the center of mass lies along the line joining the two points of contacts)
- there exists a choice of the local reference frame C_1 which simultaneously diagonalizes M, K_{r1}, and $R_{22}^T K_{r2} R_{22}$. (Note that $R_{22}^T K_{r2} R_{22}$ is the relative curvature at the second point of contact measure relative to C_1 and is hence independent of the choice of C_2).

5.2 Calculation of the stability measure

With this choice of C_1, choose C_2 such that $R_{22} = \mathrm{diag}(1, -1)$. Then, $K_{r1} = \mathrm{diag}(k_{1a}, k_{1b})$, where k_{1a} and k_{1b} are the *principle relative curvatures*. Similarly, $K_{r2} = \mathrm{diag}(k_{2a}, k_{2b})$. Furthermore, we can write $M = \mathrm{diag}(m_a, m_b)$. Computing the components of A gives

$$A_1 = \mathrm{diag}(\frac{F_n}{m_a}(r_1 - k_{1b}^{-1}), \frac{F_n}{m_b}(r_1 - k_{1a}^{-1})) \tag{16}$$

$$A_2 = \mathrm{diag}(\frac{F_n}{m_a}(r_2 - k_{2b}^{-1}), \frac{F_n}{m_b}(r_2 - k_{2a}^{-1})) \tag{17}$$

$$R_{22}^T A_1 = \mathrm{diag}(\frac{F_n}{m_a}(r_1 - k_{1b}^{-1}), -\frac{F_n}{m_b}(r_1 - k_{1a}^{-1})) \tag{18}$$

$$R_{22} A_2 = \mathrm{diag}(\frac{F_n}{m_a}(r_2 - k_{2b}^{-1}), -\frac{F_n}{m_b}(r_2 - k_{2a}^{-1})) \tag{19}$$

$$A_3 = A_4 = \mathrm{diag}(-\frac{F_n}{m_a}\kappa_f, -\frac{F_n}{m_b}\kappa_f) \tag{20}$$

$$R_{22}^T A_3 = R_{22} A_4 = \mathrm{diag}(-\frac{F_n}{m_a}\kappa_f, \frac{F_n}{m_b}\kappa_f) \tag{21}$$

The eigenvalues of A are

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0 \tag{22}$$

$$\lambda_5 = \frac{F_n}{m_a}(-\kappa_f - \sqrt{\kappa_f^2 + (d - k_{1b}^{-1} - k_{2b}^{-1})}) \tag{23}$$

$$\lambda_6 = \frac{F_n}{2m_a}(-\kappa_f + \sqrt{\kappa_f^2 + (d - k_{1b}^{-1} - k_{2b}^{-1})}) \tag{24}$$

$$\lambda_7 = \frac{F_n}{2m_b}(-\kappa_f - \sqrt{\kappa_f^2 + (d - k_{1a}^{-1} - k_{2a}^{-1})}) \tag{25}$$

$$\lambda_8 = \frac{F_n}{2m_b}(-\kappa_f + \sqrt{\kappa_f^2 + (d - k_{1a}^{-1} - k_{2a}^{-1})}) \tag{26}$$

Note that because of the four zero eigenvalues, grasps of this class can never be better than borderline stable (and hence always require active stabilization). The grasp is borderline stable if and only if

$$k_{1a}^{-1} + k_{2a}^{-1} \geq d, \quad k_{1b}^{-1} + k_{2b}^{-1} \geq d \tag{27}$$

Example 7 Let the grasped object be an ellipsoid given by the equation $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. Let the finger surfaces be flat. Let the points of contact be $(a, 0, 0)$ and $(-a, 0, 0)$. Then, the principle relative curvatures are $k_{1a} = k_{2a} = 1/b$ and $k_{1b} = k_{2b} = 1/c$, and the distance between the points of contact is $d = 2a$. Hence, the grasp is borderline stable if $b \geq a$ and $c \geq a$. This explains the intuitive assessments of Examples 1 and 3.

Example 8 Let the grasped object be a cube with sides of length l. Let the fingertips be spherical with radius R. Let the points of contact be at the centers of two opposite faces. Then, the principle relative curvatures are $k_{1a} = k_{1b} = k_{2a} = k_{2b} = 1/R$, and the distance is $d = l$. Hence, the grasp is borderline stable if $2R \geq l$.

Example 9 Consider grasping a cone of height h with spherical fingertips of radius R. Let one grasp point be the peak of the cone and the other be the center of its base. Then, $k_{1a} = k_{1b} = 1/R$, $k_{2a} = k_{2b} = \infty$, and $d = h$. Hence, the grasp is borderline stable if $R \geq h$.

5.3 Effects of the Various Factors

With the explicit representations of the eigenvalues of A derived for this class of grasps, we can see how the various factors illustrated in the examples of Section 2 affect grasp stability. As seen from Equation 27, the three factors which determine whether the grasp is naturally stable or unstable are: (1) the local geometry of the object, (2) the local geometry of the fingers, and (3) the distance between the points of contact. The first two enter Equation 27 via their contributions to the relative curvatures at the points of contact. This accounts for the intuitions of Examples 1, 2 and 3, which relate greater stability to respectively smaller object curvature, smaller finger curvature, and smaller distance between the points of contact.

The other three factors, viscoelasticity, normal forces, and moment of inertia, only determine the degree of instability of a naturally unstable grasp (and therefore, as discussed above, the difficulty of stabilizing the grasp). The eigenvalues are all proportional to the normal force F_n. Hence, the stability measure, which is the maximum of these eigenvalues, is proportional to F_n. Therefore, an increase in normal force cause unstable grasps to be more unstable. This is consistent with the intuitions of Example 4.

Half of the eigenvalues are inversely proportional to the x component of the moment of inertia, and half are inversely proportional to the y component. Holding the shape of an object fixed and varying its uniform density will yield components of the moment of intertia proportional to the density. This accounts for the intuition of Example 5 that increasing the mass of the grasped object makes unstable grasps more unstable.

To gauge the effect of viscoelasticity, observe that the only eigenvalues which can have positive real parts are λ_6 and λ_8. These both can be written in the form $\lambda(\kappa_f) = C(-\kappa_f + \sqrt{\kappa_f^2 + B})$, where κ_f is the coefficient of rolling resistance and $C > 0$ and B are independent of κ_f. For positive eigenvalues, $B > 0$. In this case, $\lambda(\kappa_f)$ is a monotonically decreasing function of κ_f. Since κ_f is a monotonically increasing function of viscoelasticity, the degree of instability of an unstable grasp is inversely related to viscoelasticity.. Note that for values of κ_f such that $\kappa_f^2 \gg B$, $\lambda(\kappa_f) \approx CB/2\kappa_f$, and hence $\lambda(\kappa_f)$ is inversely proportional to κ_f.

6 Grasping strategies

In this section, we start by defining another measure of grasp quality, contact manipulability, which is related to (and in some sense dual to) contact stability. We then examine how contact stability and contact manipulability influence the selection of grasps.

6.1 Contact Manipulability

While stability measures the ease of maintaining a grasp in its current configuration, manipulability measures the ease of changing the grasp to an arbitrary new configuration. As we distinguished between two types of grasp stability, we can also distinguish between two types of grasp manipulability. The first is joint manipulability, which measures how easily the joints can move the grasped object relative to the palm. Quantitative measures of joint manipulability are presented in [7, 19]. The second is contact manipulability, which measures how easily the contact degrees of freedom (i.e., those degrees of freedom associated with the motion of the point of contact across the surfaces of the object and the fingers) can move the grasped object relative to the palm.

Key to an understanding of contact manipulability is the fact that contact degrees of freedom are in some important ways equivalent to joints. This equivalence has been used in the kinematic analysis of multi-fingered hands (e.g., see [8]) and has been formalized in [16]. Under the restriction of rolling contact (which is what we have assumed for the grasp), the contact degrees of freedom for each point of contact are instantaneously equivalent to two revolute joints (one corresponding to rotation about the x axis and one to rotation about the y axis) [16]. When the fingers have less than six degrees of freedom (as is the case for human hands and most robotic hands), the contact degrees of freedom are needed to move the object arbitrarily relative to the palm.

The contact degrees of freedom have limits which are analogous to joint limits [16]. These *generalized joint limits* have two sources. First, the fingertips often have boundaries beyond which the points of contact should not travel. Second, the position of the points of contact on the object need to continue to form a grasp which is both spatially stable and contact stable (or at least stabilizable via active control).

Because of these limits, it is desirable to have the point of contact move as little as possible to achieve a desired rotation, or equivalently, obtain as large a rotation as possible from a given motion of the point of contact. Contact manipulability quantifies this relationship between rotation angle and movement of the point of contact. Based on Equations 2 and 3, we see that under rolling contact

$$\dot{\vec{s}}_1 = \dot{\vec{s}}_2 = K_r^{-1} \begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} \tag{28}$$

Hence, the relative curvature K_r is the map from the velocity of the point of contact to the rotational velocity.

For a two-fingered grasp, we have two contact points and hence two relative curvatures, K_{ra} and K_{rb}, each of which is the map from the velocity of the corresponding point to the corresponding rotational velocity. To evaluate the manipulability of the grasp, we need to reduce these to a single number. There are various ways to do this, but we choose the following

Definition 3 The *contact manipulability* of a grasp is the minimum of the eigenvalues of the relative curvatures (i.e., principle relative curvatures) at the contact points.

This definition implies that a grasp has contact manipulability which is equal to the manipulability of its least manipulable contact degree of freedom.

6.2 Manipulability versus stability

Grasp selection is in general a complex decision process, requiring tradeoffs between many grasp evaluation criteria based on knowledge of the task to be performed and the present state of the environment. For instance, consider grasping an egg. If the task is to transport it as safely as possible, then it is usually optimal to grasp it in the middle for maximum stability. However, if the egg is very hot, it is optimal to grasp it at the ends so as to minimize the contact area and hence the heat transfer. Furthermore, if the task is to crack the egg rather than transport it, it is optimal to grasp it at the ends to maximize the torque applied by the fingers around the crack point. Other work has examined using knowledge of the task and the environment to trade off between many different criteria (see, e.g. [10, 3]).

We focus on the two criteria of contact stability and contact manipulability. There is an inherent tradeoff between these two criteria because manipulability increases with increasing relative curvature while stability decreases. Even considering just these two criteria and restricting ourselves to two-fingered grasps, grasp selection can be complicated. It requires considering the following factors: (i) positions of the points of contact on the object, (ii) positions of the points of contact on the fingers, (iii) matching of fingers with grasp points, and (iv) orientation of the object relative to the fingers. We now discuss each of these factors and in the process derive a set of grasp selection heuristics.

Positions on the object - For grasps of the class described in Section 5, the positions of the points of contact on the object effect stability and manipulability in two ways. First, the positions of the points of contact determine the curvatures K_{1a} and K_{1b} of the object at these points. These effect stability and manipulability via their effect on the relative curvatures $K_{ra} = K_{1a} + K_{2a}$ and $K_{rb} = K_{1b} + K_{2b}$. This basic rule governs how to choose among the possibilities:

Heuristic 1 Select grasp points on the object which allow you to achieve (or come as close as possible to achieving) your goals for stability and manipulability through appropriate selection of points of contact on the fingertips and of relative orientation.

Example 10 For grasping a cube, it is usually best to select grasp points on opposite faces of the cube rather than on the edges or at the vertices. This is because the extremely high curvature at the edges and vertices means that nomatter what our choice of points of contact on the fingers the grasp will be highly unstable. (Below we discuss an exception, a case when it is better to grasp the cube at the vertices.)

The second effect is that these positions determine the distance d between the points of contact. This does not effect manipulability but effects stability as follows: increasing d decreases the stability. Hence, a second heuristic is

Heuristic 2 Consistent with Heuristic 1, minimize the distance between the points of contact.

Positions on the fingers - Human fingertips and most robotic fingertips do not have the same curvature at all points on their surfaces. In fact, human fingertips have a continuum of curvature values across their surfaces. The positions of the points of contact on the fingers determines the curvature at these points. Increasing these curvatures increases the relative curvatures at the points of contact, which increases manipulability and decreases stability. The optimal tradeoff between stability and manipulability is determined by the task, but there are some general rules we can apply. First, observe that for grasps of the class described in Section 5, the eigenvalues $\lambda_1, ..., \lambda_4$ are always equal to zero. Therefore, there is no gain in stability by decreasing the relative curvatures to make negative values $\lambda_5, ..., \lambda_8$ more negative. However, there is a loss of manipulability. This leads to

Heuristic 3 When possible, choose the points of contact on the fingers so that one of $\lambda_5, ..., \lambda_8$ is $\geq$ zero. When not possible, choose the points of contact where the curvatures are as large as possible (for human fingers, at the tips of the fingertips).

Example 11 Consider fingers such that the curvatures at all their points on their surfaces range over the set $\{kI | k_{min} \leq k \leq k_{max}\}$, and consider grasping a cube with edges of length d with grasp points on opposite faces of the cube. If $d \geq 2/k_{max}$, then we can satisfy Heuristic 3 by choosing the curvatures of the fingers to be $K_{2a} = K_{2b} = k_0 I$ where $2/d \leq k_0 \leq k_{max}$ and thus have $\lambda_6 \geq 0$ and $\lambda_8 \geq 0$. (If we do not want to rely on active stabilization, we should choose $k_0 = 2/d$ and thus have $\lambda_6 = \lambda_8 = 0$.) However, if $d < 2/k_{max}$, then there are no values of K_{2a} and K_{2b} such that $\lambda_6 \geq 0$ or $\lambda_8 \geq 0$. Heuristic 3 then says to choose $K_{2a} = K_{2b} = k_{max} I$.

A further consideration is that as d gets smaller, the surface area of the object over which the points of contact can safely move gets proportionally smaller. However, the manipulability can never exceed k_{max} for the chosen grasp points. Therefore, there is some task-dependent value d_0 such that for $d < d_0$ with the chosen grasp points there is insufficient manipulability. Then, Heuristic 1 says to choose the grasp points at the vertices rather than on the faces and accept instability for enhanced manipulability.

Note that human fingertips have curvatures whose magnitudes are bounded both above and below and are hence qualitatively similar to the fingertips of this example. Indeed, informal observations of humans grasping cubes of different sizes show behavior as described in this example. For big cubes, contact points are located on opposite faces of the cube and the flat part of the fingertips. For small cubes, contact points are located on opposite faces and the highly curved part (i.e., tip) of the fingertips. For very small cubes, contact points are located at the vertices of the cube.

Another issue in selecting the location of the contact points on the fingertips is how to distribute the relative curvature between the two points. Looking at the expressions for λ_6 and λ_8, we see that stability is a function of the sum of the relative curvatures. However, total manipulability is the minimum of the relative curvatures at each point. So,

Heuristic 4 Select the contact point on the fingertips so that the relative curvatures at the two points are as similar as possible subject to Heuristic 3 and the task goals.

Example 12 Consider the same fingers as in Example 11. Let the object curvatures at the grasp points be $K_{1a} = k_{1a}I$ and $K_{1b} = k_{1b}I$. To maximize the manipulability without making the grasp unstable, choose contact points on the finger such that $K_{2a} = (\frac{2}{d} - k_{1a})I$ and $K_{2b} = (\frac{2}{d} - k_{1b})I$ (assuming that $k_{min} \leq \frac{2}{d} - k_{1a}, \frac{2}{d} - k_{1b} \leq k_{max}$). In this case, $K_{ra} = K_{rb} = \frac{2}{d}I$.

Matching of fingers with grasp points - Sometimes the two fingers used for grasping have different shapes. For example, the thumb of a human hand is shaped differently than the forefinger. These different shapes generally yield different ranges of possible values for the curvatures of the fingers. Hence, which finger contacts the object at grasp point a and which finger contacts at grasp point b affects the range of possible stability and manipulability values achievable by varying the contact points on the fingers. So,

Heuristic 5 Choose the matching of fingers to grasp points which yields the best values of manipulability and stability after application of Heuristic 3.

Example 13 Consider grasping a cone of height h with one grasp point at the tip and the other in the middle of the base. Let the fingers be spherical but with different radii r_1 and r_2, where $r_1 < r_2$. If we match the finger with radius r_1 with the grasp point on the base (and thus the finger with radius r_2 with the tip), the grasp is borderline stable if and only if $h < r_1$. Furthermore, the manipulability of the grasp is $1/r_1$. If we match the finger with radius r_2 with the grasp point on the base, the grasp is borderline stable if and only if $h < r_2$, and the manipulability of the grasp is $1/r_2$. Hence, the matching we choose depends on the value of h. If $h \leq r_1$, then both grasps are borderline stable, and we would choose the grasp with greater manipulability, i.e. the one with r_1 matched with the base. For h very large, both grasps are highly unstable, and we would generally choose the less unstable one, i.e. the one with r_2 matched with the base. In between these extremes, there is some task-dependent cutoff r_0 such that for $h < r_0$ the greater manipulability of the r_1 match outweighs the greater stability of the r_2 match, while for $h > r_0$ the greater stability of the r_2 match outweights the greater manipulability of the r_1 match.

Observing human grasps shows qualitatively similar behavior. Humans will grasp large cones with their thumb on the base and their forefinger on the tip. However, for small cones, the forefinger will contact the base, and the thumb will contact the tip.

Orientation of the object relative to the fingers - The orientation of the object affects stability and manipulability via its effect on K_{2a} and K_{2b}, the curvatures of the fingers. These curvatures are measured relative to the local coordinate frames C_{la} and C_{lb} respectively. As the object rotates relative to the fingers so do these coordinate frames (which we recall are fixed relative to the object), which changes the values of the curvatures measured relative to the x and y axes of these frames. To maximize stability, we need to minimize the maximum of the principle relative curvatures. To maximize manipulability, we need to maximize the minimum of the principle relative curvatures. We can simultaneously achieve both these goals with the following heuristic:

Heuristic 6 Orient the object so that the major principle axes (i.e., those principle axes that correspond to the larger principle curvatures) of the object are alligned with the minor principle axes (i.e., those principle axes that correspond to the smaller principle curvatures) of the fingers. (Note that this implies that the minor principle axes of the object are alligned with the major principle axes of the fingers.)

Example 14 Consider grasping a cylinder of radius r at opposite points on a circular cross-section. Assume that we choose the local coordinate frames C_{la} and C_{lb} so that the x axes of these frames are transverse. Then, the curvatures at the grasp points are $K_{1a} = K_{1b} = \text{diag}(1/r, 0)$. Let the curvatures at the contact points on the fingers both have principle curvatures k_{minor} and k_{major}, where $k_{minor} < k_{major}$. Then, Heuristic 6 says that we should try to orient the fingers relative to the object so that $K_{2a} = K_{2b} = \text{diag}(k_{minor}, k_{major})$.

We usually see exactly this behavior when human hands grasp a cylinder with two fingers. The axis of the cylinder is perpendicular to the primary axis of the finger.

7 Conclusion

We have distinguished between two types of grasp stability, spatial grasp stability and contact grasp stability, each with a different concept of the state of a grasp. The previous work on quantifying grasp stability has focused primarily on spatial stability. Using examples, we have shown that there are differences in the stability of grasps which cannot be captured using any measure of spatial stability and hence that any full understanding of grasp stability must also involve contact stability. Therefore, we have derived a quantitative measure of contact grasp stability. To do this, we have formulated a model of the dynamics of two-fingered grasps, where the state of a grasp is the position (and velocity) of the points of contact. This model is built on concepts and results from the study of contact kinematics. The resulting equations of motion are linear; therefore, the system is stable if and only if the eigenvalues of the matrix which determines its evolution lie in the left half-plane. While the general form of this matrix is complex and does not admit a closed-form for its eigenvalues, we have calculated the eigenvalues for some examples. In these cases, the derived stability corresponds well with intuitive notions of grasp stability. We have then introduced the concept of contact manipulability, a grasp evaluation criterion which is dual to contact stability insofar as increasing relative curvature at the points of contact will generally increase manipulability but decrease stability. Choosing a grasp which gives an optimal tradeoff between stability and manipulability requires an appropriate selection of (i) grasp points on the object, (ii) contact points on the fingertips, (iii) matching of grasp points with fingers, and (iv) orientation of the object relative to the fingers. We have discussed the role of each of these factors with respect to the stability versus manipulability tradeoff.

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